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Subject Name: **Theory of machines & mechanics**

Subject Code: **ME-3003**

Semester: **3rd**

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Unit-III

Gears: Classification of gears, Helical, Spiral, Bevel, Worm and Spur Gear, Spur Gear Terminology, Law of gearing, Tooth profiles, , velocity of sliding, Path of contact, Arc of contact, Contact Ratio, Interference and Undercutting, , Conjugate action, Numerical problems based on above topics
Gear Trains: Simple, compound, reverted and epicyclic gear trains. Velocity ratio and torque calculation in gear trains

Classification of gears:

Classification about these **types of gears** discussed below.

1. Gears for Parallel Shafts:

The motion between parallel shafts is same as to the rolling of two cylinders. Gears under this category are the following:

1.1 Spur Gears:

Straight Spur gears are the simplest form of gears having teeth parallel to the gear axis. The contact of two teeth takes place over the entire width along a line parallel to the axes of rotation. As gear rotate, the line of contact goes on shifting parallel to the shaft.

Fig 3.1 Spur Gears

1.2 Helical Gears:

In helical gear teeth are part of helix instead of straight across the gear parallel to the axis. The mating gears will have same helix angle but in opposite direction for proper mating. As the gear rotates, the contact shifts

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along the line of contact in involute helicoids across the teeth.

Fig 3.2 Helical Gear

1.3. Herringbone Gears:

Herringbone gears are also known as **Double Helical Gears**. Herringbone gears are made of two helical gears with opposite helix angles, which can be up to 45 degrees.

Fig 3.3 Herringbone gears

1.4. Rack and Pinion:

In these gears the spur rack can be considered to be spur gear of infinite pitch radius with its axis of rotation placed at infinity parallel to that of pinion. The pinion rotates while the rack translates.

Fig 3.4 Rack and Pinion

2. Gears for Intersecting Shafts:

The motion between two intersecting shafts is equivalent to the rolling of two cones. The gears used for intersecting shafts are called bevel gears. Gears under this category are following:

2.1 Straight Bevel Gears:

Straight bevel gears are provided with straight teeth, radial to the point of intersection of the shaft axes and vary in cross section through the length inside generator of the cone. Straight Bevel Gears can be seen as modified version of straight spur gears in which teeth are made in conical direction instead of parallel to axis.

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Fig 3.5 Straight bevel gears

2.2 Spiral Bevel Gears:

Bevel gears are made with their teeth are inclined at an angle to face of the bevel. Spiral gears are also known as helical bevels.

Fig 3.6 Spiral Bevel Gears

3. Gears for Skew Shafts:

The following gears are used to join two non-parallel and non-intersecting shafts.

3.1 Hypoid Gears:

The Hypoid Gears are made of the frusta of hyperboloids of revolution. Two matching hypoid gears are made by revolving the same line of contact, these gears are not interchangeable.

Fig 3.6 Hypoid Gears

3.2 Worm Gears:

The Worm Gears are used to connect skewed shafts, but not necessarily at right angles. Teeth on worm gear are cut continuously like the threads on a screw. The gear meshing with the worm gear is known as worm wheel and combination is known as worm and worm wheel.

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Fig 3.6 Worm Gears

Spur Gear Terminology:

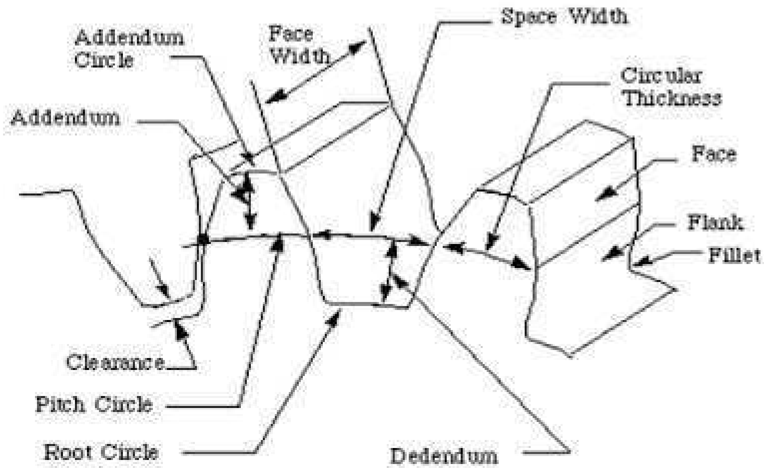

Fig 3.7 Spur Gear Terminology

➢ **Pitch surface**: The surface of the imaginary rolling cylinder (cone, etc.) that the toothed gear may be considered to replace.

➢ **Pitch circle**: A right section of the pitch surface.

➢ **Addendum circle**: A circle bounding the ends of the teeth, in a right section of the gear.

➢ **Root (or dedendum) circle**: The circle bounding the spaces between the teeth, in a right section of the gear.

➢ **Addendum**: The radial distance between the pitch circle and the addendum circle.

➢ **Dedendum**: The radial distance between the pitch circle and the root circle.

➢ **Clearance**: The difference between the dedendum of one gear and the addendum of the mating gear.

➢ **Face of a tooth**: That part of the tooth surface lying outside the pitch surface.

➢ **Flank of a tooth**: The part of the tooth surface lying inside the pitch surface.

➢ **Circular thickness** (also called the **tooth thickness**): The thickness of the tooth measured on the pitch circle. It is the length of an arc and not the length of a straight line.

➢ **Tooth space**: The distance between adjacent teeth measured on the pitch circle.

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- **Backlash**: The difference between the circle thickness of one gear and the tooth space of the mating gear.
- Backlash =Space width – Tooth thickness
- **Circular pitch** p: The width of a tooth and a space, measured on the pitch circle.
- **Diametral pitch** P: The number of teeth of a gear per inch of its pitch diameter. A toothed gear must have an integral number of teeth. The circular pitch, therefore, equals the pitch circumference divided by the number of teeth. The diametral pitch is, by definition, the number of teeth divided by the pitch diameter.
- **Module** m: Pitch diameter divided by number of teeth. The pitch diameter is usually specified in inches or millimeters; in the former case the module is the inverse of diametral pitch.
- **Fillet**: The small radius that connects the profile of a tooth to the root circle.
- **Pinion**: Smaller of any pair of mating gears. Larger of the pair is called simply the gear.
- **Velocity ratio**: The ratio of the number of revolutions of the driving (or input) gear to the number of revolutions of the driven (or output) gear, in a unit of time.
- **Pitch point**: The point of tangency of the pitch circles of a pair of mating gears.
- **Common tangent**: The line tangent to the pitch circle at the pitch point.
- **Base circle**: An imaginary circle used in involute gearing to generate the involutes that form the tooth profiles.
- **Line of Action or Pressure Line:** The force, which the driving tooth exerts at point of contact of the two teeth. This line is also the common tangent at the point of contact of the mating gears and is known as the line of action or the pressure line. The component of the force along the common tangent at the p point is responsible for the power transmission.

The component of the force perpendicular to the common tangent through the pitch point produces the required thrust.

Pressure Angle or Angle of Obliquity (ϕ): The angle between pressure line and the common tangent to the pitch circles is known as the pressure angle or the angle of obliquity.

For more power 'transmission and lesser pressure on the bearing pressure angle must be kept small. Standard pressure angles arc and 25°. Gears with 14.5° pressure angles have become almost obsolete.

- **Path of Contact or Contact Length:** Locus of the point of contact between two mating teeth from the beginning of engagement to the end is known as the path of contact or the contact length. It is CD in the figure. Pitch point P is always one point on the path of contact. It can be subdivided as follows:
- **Path of Approach:** Portion of the path of contact from the beginning of engagement to the pitch point, i.e. the length CP.
- **Path of Recess:** Portion of the path of contact from the pitch point to the end of engagement i.e. length PD.
- **Arc of Contact:** Locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact in fig. , APB or EPF is the arc of contact. It has also been divided into sub-portions.
- **Arc of Approach:** It is the portion of the arc of contact from the beginning of engagement to the pitch point, i.e. length AP or EP.
- **Arc of Recess:** Portion of the arc of contact from the pitch point to the end of engagement is the arc of recess i.e. length PB or PF.
- **Angle of Action (δ):** It is the angle turned by a gear from the beginning of engagement to the end of engagement of a pair of teeth i.e. the angle turned by arcs of contact of respective gear wheels. Similarly, angle of approach (a) and angle of recess (β) can be defined.

S=a+ β

Circular Pitch = π ÷ Diametral Pitch
Diametral Pitch = π ÷ Circular Pitch
Pitch Diameter = Teeth ÷ Diametral Pitch

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Pitch Diameter = Teeth × Circular Pitch ÷ pi
Center Distance = (Teeth on Pinion + Teeth on Gear) ÷ (2 × Diametral Pitch)
Center Distance = (Teeth on Pinion + Teeth on Gear) × Circular Pitch ÷ (2 × pi)
Diametral Pitch = (Teeth on Pinion + Teeth on Gear) ÷ (2 × Center Distance)
Circular Pitch = Center Distance × 2 × π ÷ (Teeth on Pinion + Teeth on Gear)
Teeth = Pitch Diameter × Diametral Pitch
Teeth = Pitch Diameter × π ÷ Circular Pitch
Base Circle Diameter = Pitch Diameter × Cosine (Pressure Angle)

Law of gearing: Law of gearing stats that "the angular velocity ratio of all gears of a meshed gear system must remain constant also the common normal at the point of contact must pass through the pitch point."

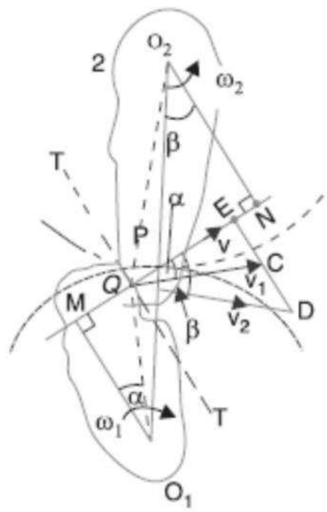

Fig 3.8 Law of Gearing

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. 3.8. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure 3.8.

Let TT be the common tangent and M N be the common normal to the curves at the point of contact Q. From the centers O_1 and O_2 , draw O_1M and O_2N perpendicular to M N. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1 and v_2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal M N must be equal.

$V_1 \cos \alpha = v_2 \cos \beta$

$(\omega_1 \text{ X } O_1Q) \cos \alpha = (\omega_2 \text{ X } O_2Q) \cos \beta$

$(\omega_1 \text{ X } O_1Q)\ O_1M/O_1Q = (\omega_2 \text{ X } O_2Q)\ O_2N/O_2Q$

$\omega_1 / \omega_2 = O_2N/ O_1M$

Also from similar trianglesO1MP and O2NP,

$O_2N/O_1M = O_2P/O_1P$

Combining above equations we have

$\omega_1 / \omega_2 = O_2N/O_1M = O_2P/O_1P$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centers O_1 and O_2, or the common normal to the two surfaces at the point of contact Q intersects the line of centers at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be

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the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

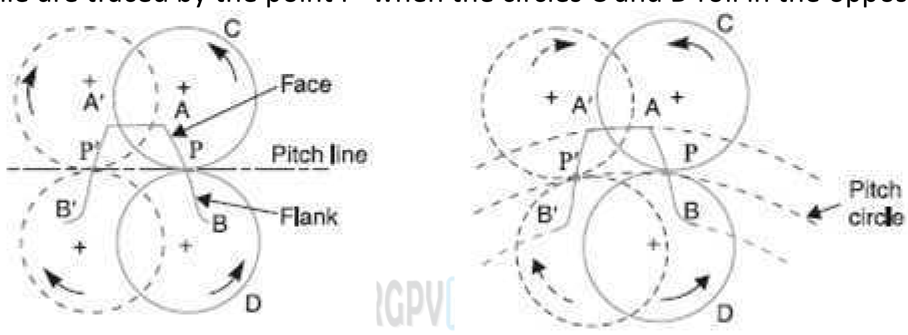
Tooth profiles:

In actual practice following are the two types of teeth commonly used:

1. Cycloidal teeth:

A **cycloid** is the curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line. When a circle rolls without slipping on the outside of a fixed circle, the curve traced by a point on the circumference of a circle is known as **epicycloids**. On the other hand, if a circle rolls without slipping on the inside of a fixed circle, then the curve traced by a point on the circumference of a circle is called **hypocycloid**.

In Fig. 3.9 (a), the fixed line or pitch line of a rack is shown. When the circle C rolls without slipping above the pitch line in the direction as indicated in Fig. 3.9 (a), then the point P on the circle traces epicycloids PA. This represents the face of the cycloidal tooth profile. When the circle D rolls without slipping below the pitch line, then the point P on the circle D traces hypo-cycloid PB, which represents the flank of the cycloidal tooth. The profile BPA is one side of the cycloidal rack tooth. Similarly, the two curves P' A' and P'B' forming the opposite side of the tooth profile are traced by the point P' when the circles C and D roll in the opposite directions.



(a) (b)

Fig 3.9 Construction of cycloidal teeth of a gear

In the similar way, the cycloidal teeth of a gear may be constructed as shown in Fig. 3.9 (b). The circle C is rolled without slipping on the outside of the pitch circle and the point P on the circle C traces epicycloids PA, which represents the face of the cycloidal tooth. The circle D is rolled on the inside of pitch circle and the point P on the circle D traces hypo-cycloid PB, which represents the flank of the tooth profile. The profile BPA is one side of the cycloidal tooth. The opposite side of the tooth is traced as explained above.

The construction of the two mating cycloidal teeth is shown in Fig. 3.10. A point on the circle D will trace the flank of the tooth T_1 when circle D rolls without slipping on the inside of pitch circle of wheel 1 and face of tooth T_2 when the circle D rolls without slipping on the outside of pitch circle of wheel 2. Similarly, a point on the circle C will trace the face of tooth T_1 and flank of tooth T_2. The rolling circles C and D may have unequal diameters, but if several wheels are to be interchangeable, they must have rolling circles of equal diameters.

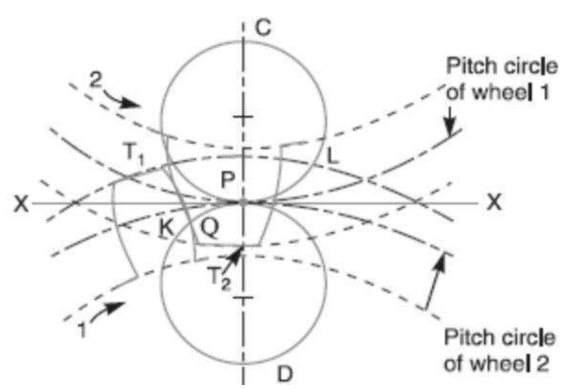


Fig 3.10 Construction of two mating cycloidal teeth

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A little consideration will show that the common normal XX at the point of contact between two cycloidal teeth always passes through the pitch point, which is the fundamental condition for a constant velocity ratio.

2. Involute teeth

An involute of a circle is a plane curve generated by a point on a tangent, which rolls on the circle without slipping or by a point on a taut string which in unwrapped from a reel as shown in Fig. 3.11. In connection with toothed wheels, the circle is known as base circle. The involute is traced as follows:

Let A be the starting point of the involute. The base circle is divided into equal number of parts e.g. AP_1, P_1P_2, and P_2P_3 etc. The tangents at P_1, P_2, P_3 etc. are drawn and the length P_1A_1, P_2A_2, P_3A_3 equal to the arcs AP_1, AP_2 and AP_3 are set off. Joining the points A, A_1, A_2, A_3 etc. we obtain the involute curve A R. A little consideration will show that at any instant A_3, the tangent A_3T to the involute is perpendicular to P_3A_3 and P_3A_3 is the normal to the involute. In other words, **normal at any point of an involute is a tangent to the circle.**

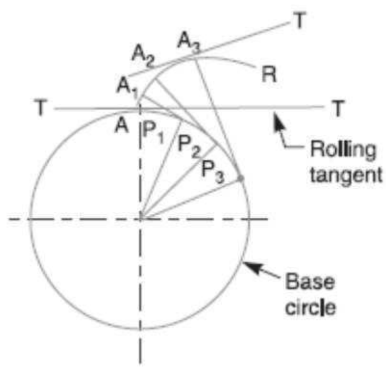
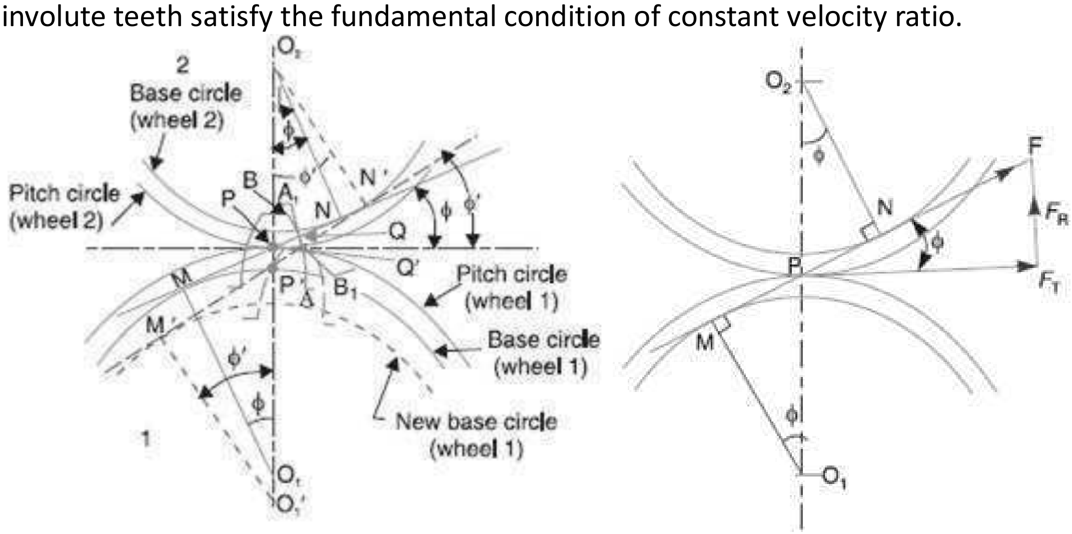


Fig 3.11 Construction of involute

Now, let O_1 and O_2 be the fixed centers of the two base circles as shown in Fig. 3.12 (a). Let the corresponding involutes A B and A_1B_1 be in contact at point Q. MQ and NQ are normal to the involutes at Q and are tangents to base circles. Since the normal of an involute at a given point is the tangent drawn from that point to the base circle, therefore the common normal M N at Q is also the common tangent to the two base circles. We see that the common normal M N intersects the line of centers O_1O_2 at the fixed point P (called pitch point). Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.



(a) (b)

Fig 3.12 Involute teeth

From similar triangles O_2NP and O_1MP,

$O_1M/O_2N = O_1P/O_2P = \omega_2 / \omega_1$

From above equation, which determines that the ratio of the radii of the two base circles. The radii of the base

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circles is given by

$O_1 M = O_1 P \cos \phi$, and $O_2 N = O_2 P \cos \phi$

Also the centre distance between the base circles,

$O_1O_2 = O_1P + O_2P = O_1M/\cos \phi + O_2N/ \phi = (O_1M + O_2N) / \phi$

Where ϕ = pressure angle or the angle of obliquity. It is the angle which the common normal to the base circles (i.e. MN) makes with the common tangent to the pitch circles.

When the power is being transmitted, the maximum tooth pressure (neglecting friction at the teeth) is exerted along the common normal through the pitch point. This force may be resolved into tangential and radial or normal components. These components act along and at right angles to the common tangent to the pitch circles.

If F is the maximum tooth pressure as shown in Fig. 3.12 (b), then

Tangential force, $F_T = F \cos \phi$

And radial or normal force, $F_R = F \sin \phi$.

∴ Torque exerted on the gear shaft = $F_T \times r$, where r is the pitch circle radius of the gear.

Velocity of Sliding of Teeth:

The sliding between a pair of teeth in contact at Q occurs along the common tangent TT to the tooth curves as shown in Fig. **The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact**.

The velocity of point Q, considered as a point on wheel 1, along the common tangent TT is represented by EC. From similar triangles QEC and O_1MQ,

$EC/MQ = v/O_1Q = \omega_1$ Or $EC = \omega_1 . MQ$

Similarly, the velocity of point Q, considered as a point on wheel 2, along the common tan-gent TT is represented by ED. From similar triangles QCD and O_2 NQ,

$ED/QN = v_2/O_2Q = \omega_2$ or $ED = \omega_2 . QN$

v_s = Velocity of sliding at Q

$v_s = ED – EC = (\omega_2 . QN) - (\omega_1 . MQ)$

$v_s = \omega_2 (QP + PN) – \omega_1 (MP – QP)$

$v_s = (\omega_1 + \omega_2) QP + \omega_2 . PN - \omega_1 . MP$

Since

$\omega_1 / \omega_2 = O_2P/O_1P = PN/MP$ or $\omega_1 . MP = \omega_2 . PN$

Since therefore equation becomes

$V_s = (\omega_1 + \omega_2) QP$

Length of Path of Contact

Consider a pinion driving the wheel as shown in Fig. 3.13. When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at K (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at L (outer end of the tooth face on the pinion or on the flank near the base circle of wheel). M N is the common normal at the point of contacts and the common tangent to the base circles. The point K is the intersection of the addendum circle of wheel and the common tangent. The point L is the intersection of the addendum circle of pinion and common tangent.

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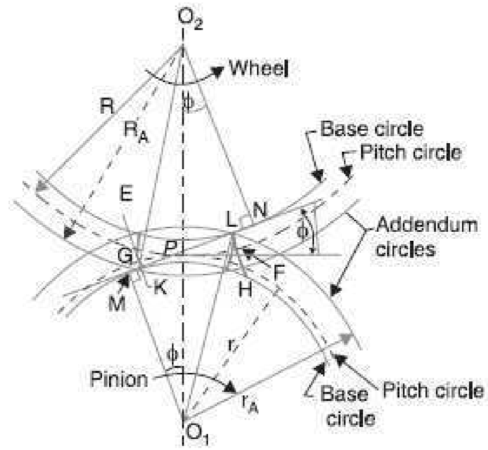

Fig. 3.13 Length of path of contact

The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion. Thus the length of path of contact is K L which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as **path of approach** and the part of the path of contact PL is known as **path of recess.**

Let $r_A = O_1L$ = Radius of addendum circle of pinion,

$R_A = O_2K$ = Radius of addendum circle of wheel,

$r = O_1P$ = Radius of pitch circle of pinion, and

$R = O_2P$ = Radius of pitch circle of wheel.

From Fig., we find that radius of the base circle of pinion,

$O_1M = O_1P \cos \phi = r \cos \phi$ and radius of the base circle of wheel,

$O_2N = O_2P \cos \phi = R \cos \phi$ Now from right angled triangle $O_2K N$,

$KN = [(O_2 K)^2 - (O_2 N)^2]^{1/2} = [R_A^2 - R^2 \cos^2 \phi]^{1/2}$

And $PN = O_2 P \sin \phi = R \sin \phi$

∴ Length of the part of the path of contact, or the path of approach,

$KP = KN - PN = [R_A^2 - R^2 \cos^2 \phi]^{1/2} - R \sin \phi$

Similarly from right angled triangle $O_1M L$,

And $ML = [(O_1L)^2 - (O_1M)^2]^{1/2} = [r_A^2 - r^2 \cos^2 \phi]^{1/2}$

And $MP = O_1 P \sin \phi = r \sin \phi$

∴ Length of the part of the path of contact, or path of recess,

$PL = ML - MP = [r_A^2 - r^2 \cos^2 \phi]^{1/2} - r \sin \phi$

∴ Length of the path of contact,

$KL = KP + PL = [R_A^2 - R^2 \cos^2 \phi]^{1/2} + [r_A^2 - r^2 \cos^2 \phi]^{1/2} - (R + r) \sin \phi$

Length of Arc of Contact

We have already defined that the arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Fig. 3.13, the arc of contact is EPF or GPH. Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as **arc of approach** and the arc PH is called **arc of recess**. The angles subtended by these arcs at O_1 are called **angle of approach** and **angle of recess** respectively.

We know that

The length of the arc of approach (arc GP) = Length of path of approach / cos ø = KP / cos ø

And the length of the arc of recess (arc PH) = Length of Path of recess / cos ø = PL / cos ø

Since the length of the arc of contact GPH is equal to the sum of the length of arc of approach and arc of recess, therefore,

Length of the arc of contact = arc GP + arc PH = KP/ cos ø + PL/ cos ø = KL/ cos ø

= Length of path of contact / cos ø

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Contact Ratio (or Number of Pairs of Teeth in Contact)
The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch.** Mathematically,

Contact ratio or number of pairs of teeth in contact
= Length of the arc of contact / p_C
Where p_c = Circular pitch = π m, and m = Module.

Interference in Involute Gears
Fig. 3.14 shows a pinion with centre O_1, in mesh with wheel or gear with centre O_2. MN is the common tangent to the base circles and K L is the path of contact between the two mating teeth.

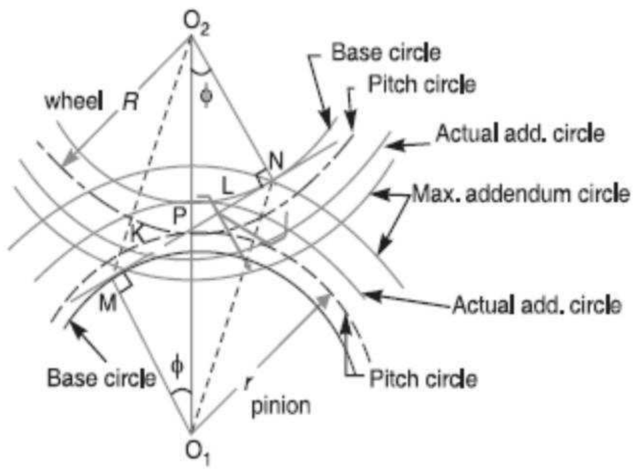

Fig. 3.14 Interference in involute gears

A little consideration will show that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of tooth on the wheel. This effect is known as **interference,** and occurs when the teeth are being cut. In brief, **the phenomenon when the tip of tooth undercuts the root on its mating gear is known as interference.**
Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion. The points M and N are called **interference points.** Obviously, interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and of the wheel is O_2M.
From the above discussion, we conclude that the interference may only be avoided, if the point of contact between the two teeth is always on the involute profiles of both the teeth. In other words, **interference may only be prevented, if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.**
When interference is just avoided, the maximum length of path of contact is M N when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M respectively as shown in Fig. In such a case,
Maximum length of path of approach,
$MP = r \sin \phi$
And maximum length of path of recess,
$PN = R \sin \phi$
∴ Maximum length of path of contact,
$MN = MP + PN = r \sin \phi + R \sin \phi = (r + R) \sin \phi$
And maximum length of arc of contact
$= (r + R) \sin \phi / \cos \phi = (r + R) \tan \phi$

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Gear Trains

Introduction

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains

Following are the different types of gear trains, de-pending upon the arrangement of wheels:

1. Simple gear train,
2. Compound gear train,
3. Reverted gear train, and
4. Epicyclic gear train.

In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

Simple Gear Train

When there is only one gear on each shaft, as shown in Fig. 3.15 it is known as simple gear train. The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Fig. 3.15 (a). Since the gear 1 drives the gear 2 and therefore gear 1 is called the driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

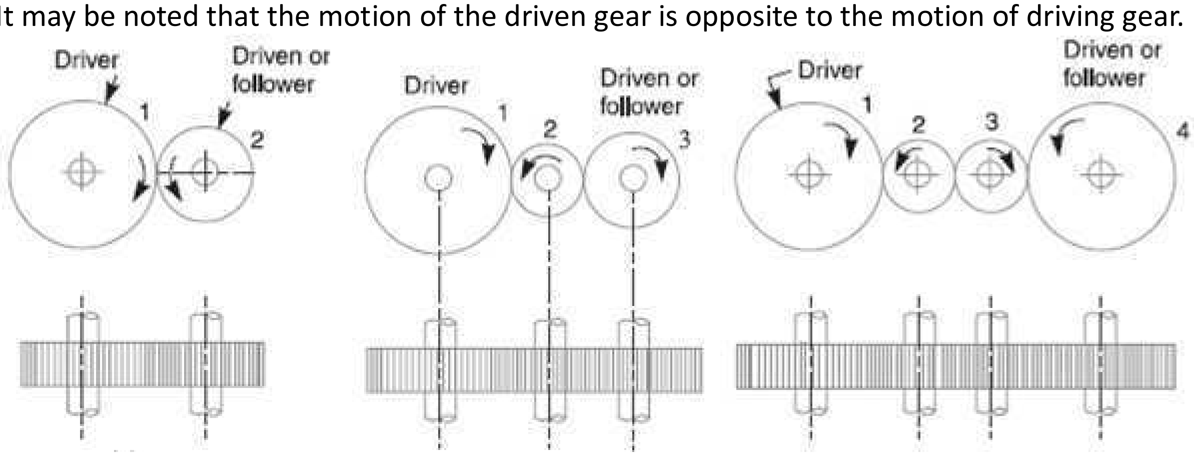

Fig. 3.15 Simple gear train

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Let N_1 = Speed of gear 1(or driver) in r.p.m.,
N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,
T_1 = Number of teeth on gear 1, and
T_2 = Number of teeth on gear 2.
Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore
Speed ratio = $N_1/N_2 = T_2/T_1$
It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train. Mathematically,
Train value = $N_2/N_1 = T_1/T_2$
From above, we see that the train value is the reciprocal of speed ratio.

Compound Gear Train
When there is more than one gear on a shaft, as shown in Fig. 3.16 it is called a compound train of gear. But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in Fig. 3.16.

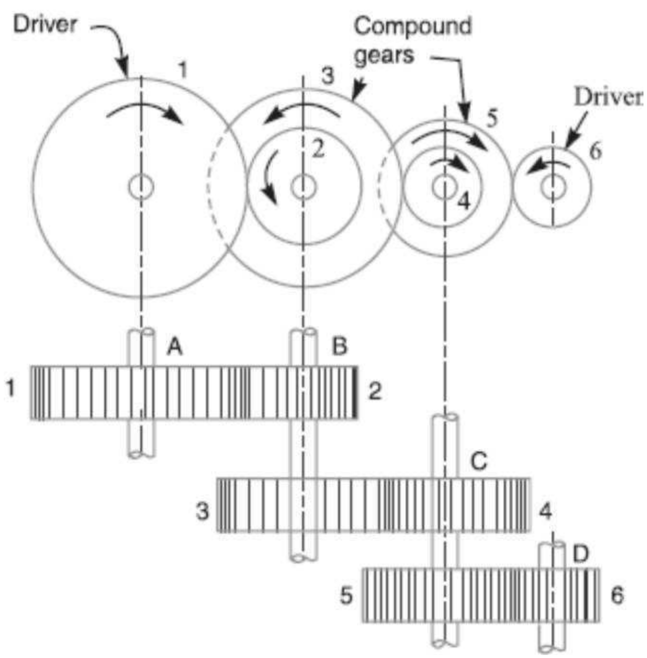

Fig. 3.16 Compound Gear Train

In a compound train of gears, as shown in Fig. 3.16, the gear 1 is the driving gear mounted on shaft A and gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.
Let N_1 = Speed of driving gear 1,
T_1 = Number of teeth on driving gear 1,
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and $T_2, T_3..., T_6$ = Number of teeth on respective gears.
Since gear 1 is in mesh with gear 2, therefore its speed ratio is
$N_1/N_2 = T_2/T_1$
Similarly, for gears 3 and 4, speed ratio is
$N_3/N_4 = T_4/T_3$
And for gears 5 and 6, speed ratio is
$N_5/N_6 = T_6/T_5$
The speed ratio of compound gear train is obtained by multiplying the above equations,

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$N_1/N_2 \times N_3/N_4 \times N_5/N_6 = T_2/T_1 \times T_4/T_3 \times T_4/T_3$
Or
$N_1/N_6 = (T_2 \times T_4 \times T_6) / (T_1 \times T_3 \times T_5)$
The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train as shown in Fig. 3.17.

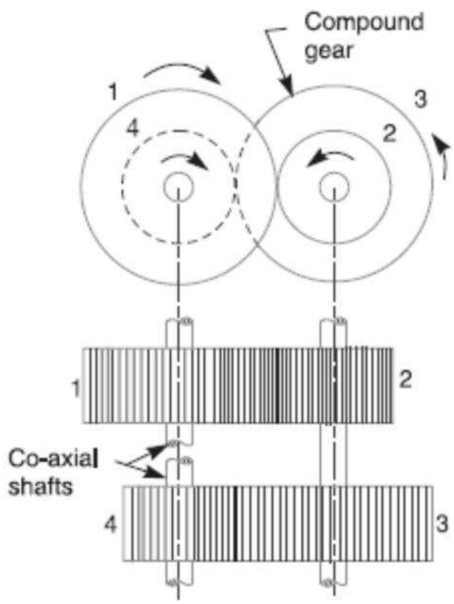

Fig. 3.17 Reverted Gear Train

T_1 = Number of teeth on gear 1
r_1 = Pitch circle radius of gear 1, and
N_1 = Speed of gear 1 in r.p.m.
Similarly,
T_2, T_3, T_4 = Number of teeth on respective gears,
r_2, r_3, r_4 = Pitch circle radii of respective gears, and
N_2, N_3, N_4 = Speed of respective gears in r.p.m.
Since the distance between the centers of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore
$r_1 + r_2 = r_3 + r_4$ ... Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.
$T_1 + T_2 = T_3 + T_4$
Speed ratio = Product of number of teeth on drivens / Product of number of teeth on drivers
$N_1/N_4 = (T_2 \times T_4) / (T_1 \times T_3)$

Epicyclic Gear Train

In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig. 3.18 where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2, about

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which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice-versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O_1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member is known as epicyclic gear trains (epi means upon and cyclic means around). The epicyclic gear trains may be simple or compound.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

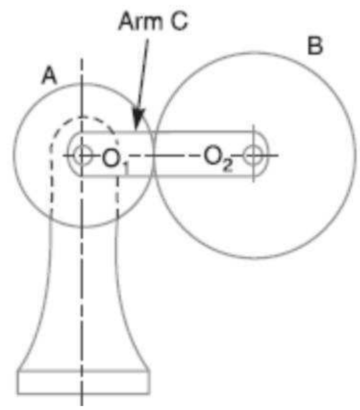

Fig. 3.18 Epicyclic gear train

Velocity Ratio of Epicyclic Gear Train

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method, and

2. Algebraic method

These methods are discussed, in detail, as follows:

1. Tabular method. Consider an epicyclic gear train as shown in Fig. 3.18.

Let T_A = Number of teeth on gear A, and

T_B = Number of teeth on gear B.

First of all, let us suppose that the arm is fixed. Therefore the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make $(– T_A / T_B)$ revolutions. This statement of relative motion is entered in the first row of the table.

Secondly, if the gear A makes + x revolutions, then the gear B will make $– x \times T_A / T_B$ revolutions. This statement is entered in the second row of the table. In other words, multiply the each motion (entered in the first row) by x.

Thirdly, each element of an epicyclic train is given + y revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1	Arm fixed-gear A rotates through + 1 revolution i.e. anticlockwise	0	+1	$-T_A/T_B$
2	Arm fixed-gear A rotates through + x revolutions	0	+x	$- x \times T_A/T_B$
3	Add + y revolutions to all elements	+y	+y	+y
4	Total motion	+y	x + y	$y - x \times T_A/T_B$

A little consideration will show that when two conditions about the motion of rotation of any two elements

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are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

2. Algebraic method. In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train viz. some element is fixed and the other has specified motion. These two conditions are sufficient to solve all the equations; and hence to determine the motion of any element in the epicyclic gear train.

Let the arm C be fixed in an epicyclic gear train as shown in Fig. 3.18 Therefore speed of the gear A relative to the arm C = $N_A - N_C$

And speed of the gear B relative to the arm C,

$= N_B - N_C$

Since the gears A and B are meshing directly, therefore they will revolve in **opposite** directions.

$(N_B - N_C) / (N_A - N_C) = - T_A / T_B$

Since the arm C is fixed, therefore its speed, $N_C = 0$

$N_B / N_A = - T_A / T_B$

If the gear A is fixed, then $N_A = 0$.

$(N_B - N_C) / (0 - N_C) = - T_A / T_B$

$N_B / N_C = 1 - T_A / T_B$

Torque ratio

A gear train can be analyzed using the principle of virtual work to show that its torque ratio, which is the ratio of its output torque to its input torque, is equal to the gear ratio, or speed ratio, of the gear train.

This means that the input torque T_A applied to the input gear G_A and the output torque T_B on the output gear G_B are related by the ratio

$R = T_B / T_A$

Where R = the gear ratio of the gear train.

The torque ratio of a gear train is also known as its mechanical advantage

$M_A = T_B / T_A$

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